Missing Lorenz-boosted Circles-in-the-sky

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A topologically finite universe, smaller than the observable horizon, will have circles-in-the-sky: pairs of circles around which the temperature fluctuations in the cosmic microwave background are correlated. The circles occur along the intersection of copies of the spherical surface of last scattering. For any observer moving with respect to the microwave background, the circles will be deformed into ovals. The ovals will also be displaced relative to the direction they appear in a comoving frame. The displacement is the larger of the two effects. In a Lorenz boosted frame, the angular displacement of a point on the surface of last scattering relative to the comoving frame is proportional to the velocity. For the Earth's motion, the effect is on the order of 0.14^o at the very worst. If we live in a small universe and are looking for an identical copy of a spot in the sky, it may be displaced by as much as 0.14^o from where we expect. This can affect all pattern based searches for the topology of the universe. In particular, high-resolution searches for circle pairs could be off by this much.

The earth is nearly comoving with the expansion of the universe but not quite. The entire galaxy moves with respect to the cosmic microwave background (CMB) at a speed $\beta = v/c = 1.23\times 10^{-3}$ relative to the speed of light. The motion of the earth creates a large dipole-fluctuation in the CMB temperature since the universe looks hotter in the direction of our motion than in the opposite direction. The dipole is subtracted from data sets such as those from COBE [1] and WMAP [2] to remove non-cosmological contributions to the maps. Subtracting the dipole alone does not correct the map for any distortions in the shape or location of features in the sky. Ordinarily this doesn't matter in the least since the universe is assumed to be homogeneous and isotropic. Analysts are interested in angular averages over the sky, not the precise location of a given hot or cold spot.

In a finite universe, homogeneity and isotropy cannot be assumed globally since topological identifications nearly always break these symmetries (see reviews [3, 4, 5, 6]). A search of the CMB data for evidence of a finite universe relies on detailed information on the shape and location of features in the sky. These pattern-based searches [7, 8, 9] all ignore the motion of the earth with respect to the CMB. As shown below, there is a small effect, yet none-the-less relevant to pattern-based searches, due to the earth's motion.

In a topologically compact space, there are a preferred set of observers for whom the volume of space is smallest [10, 11]. In the preferred frame, space is topologically identified but there is no mixing of the time component in the identification rules. This frame naturally coincides with the comoving frame and the preferred observers are at rest with respect to the expansion of the universe [11]. For the sake of argument, consider one such hypothetical observer, C, completely at rest with respect to the CMB and living in a flat spacetime. Working in his comoving frame, last scattering occurred at a time $\eta_0 - \eta_*$ in the past where η_0 is the age of the universe today and η_* is the time of last scattering. For the sake of argument we will take last scattering to happen at an instant. Since our experiments measuring the microwave background are effectively instantaneous relative to the age of the universe, all of the light collected traveled exactly the same distance in all directions thereby defining a spherical surface of radius $\eta_0 - \eta_* = r$. All the CMB light C observes originated on this surface of last scattering.

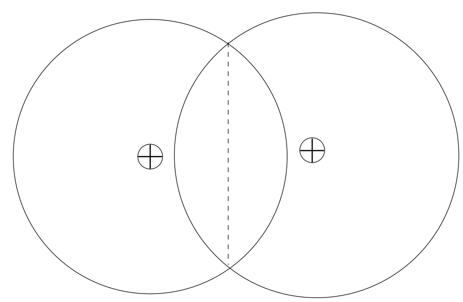

FIG. 1: The surface of last scattering intersects with itself in a finite universe smaller than the diameter of the surface. The self-intersection can most easily be visualized in a tiling picture, such as the above where a two-dimensional slice through space is shown. The intersection of the two spheres occurs along the dotted-line circle. When an observer at rest with respect to the expansion looks to the right, he sees temperature fluctuations along the dotted-line. When he looks to the left he sees identical temperature fluctuations along the dotted-line. Therefore he measures a correlated pair of circles, one to the right, the other to the left.

A finite universe looks like an infinite universe tiled with an infinite number of copies of the fundamental space. In each tile is an identical copy of the earth and each image of the earth is encapsulated by an image of the surface of last scattering. Some of these copies of the surface of last scattering will be near enough to each other in the tiling to overlap. The overlap occurs along a circle so that observer C will see identical variations in the temperature fluctuations occurring along circle pairs [7] as illustrated in figures 1 and 2. These circles-in-the-sky, as they were coined when found in Ref. [7], are particularly important observationally since they occur in any topologically compact space and in principle can be

detected without any prior assumption about the shape and size of space, as long as the universe is smaller than the surface of last scattering.

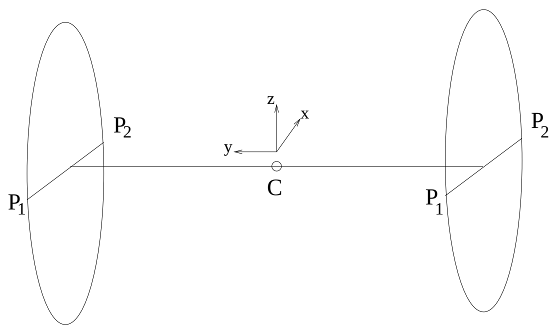

FIG. 2: Another view of the correlated circle pair of figure 1. C sees one circle in front of him and one behind him. In general, circle pairs may be out of phase and not face-to-face.

In principle, a discovery of circles-in-the-sky would be an unambiguous and definitive observation of topology. In practice, there are impediments to making such an observation. Lensing of photons along the line of sight from decoupling until today can deflect fluctuations off the circle or damage the correlation between circle pairs, as can reionization or an integrated Sachs-Wolfe contribution. Other important effects include a finite thickness to the surface of last scattering as well as Doppler effects on small scales. Matter oscillations can Doppler shift the CMB in a directionally dependent way.(In Refs. [9, 12], the resilience of pattern-based searches to at least some of these obstacles is confirmed in numerical experiments.) In this article we add to this list the small, but still present, effect of a Lorenz deformation of the circles into displaced ovals.

To derive the distortion consider an observer, O, who moves with speed β relative to the CMB. At the time the satellites measure the CMB, observer O coincides with observer C.

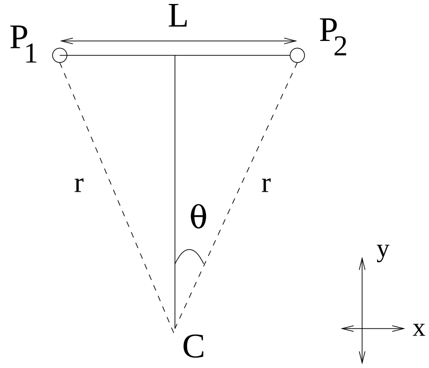

FIG. 3: Two points on opposite sides of a circle-in-the-sky from figure 2. The z-direction is suppressed.

To prove that circles deform to displaced ovals, consider two points P_1 and P_2 on opposite sides of a circle of radius $L/2$ at rest with respect to C as measured in the C frame. The two points P_1 and P_2 lie along the axis of motion of observer O. We take the motion to be along the x-axis. The geometry for C is shown in figure 3. The emission of last scattered photons from points P_1 and P_2 are two events that occur simultaneously and a distance L apart so that $\Delta\eta = \eta_2 - \eta_1 = 0$ and $\Delta x = x_2 - x_1 = L$. The space interval, $\Delta x' = x'_2 - x'_1$, and time interval, $\Delta\eta' = \eta'_2 - \eta'_1$, of the events P_1 and P_2 as measured by the observer O are given by the Lorenz transformation,

$$\begin{pmatrix} \Delta\eta' \\ \Delta x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \Delta\eta \\ \Delta x \end{pmatrix} \quad , \tag{1}$$

with $\gamma = (1-\beta^2)^{-1/2}$ so that

$$\begin{aligned} \Delta\eta' &= -\gamma\beta L \\ \Delta x' &= \gamma L \quad . \end{aligned} \tag{2}$$

In words, observer O does not believe that the emission of last scattered light occurs simultaneously but instead observes P_2 emit light before P_1. O also observes the distance between the two points to be larger than in the rest frame by the factor γ. The elongation occurs only along the direction of motion. The perpendicular axis is still observed to have length L and the circle is deformed into an ellipses with long axis γL and short axis L.

Another way to derive this result is to begin with the Lorenz contraction. Observers in the O frame would measure the intrinsic distance between opposite points on the circle to be Lorenz contracted to L/γ. They would come to this conclusion by performing the following experiment. Traveling in a rocket at speed β, their spaceship coincides with P_1 at time $\bar{\eta}'_1$ as shown in figure 4. The location at which this happens in their frame is the center of the rocket which they take to be the origin so $\bar{x}'_1 = 0$. Some time later the ship coincides with point P_2 at time $\bar{\eta}'_2$. Again the location at which this happens is the center of the rocket, namely $\bar{x}'_2 = 0$. Now, they see P_1 and P_2, pass by their windows traveling at speed β and so the time elapsed $\bar{\eta}'_2 - \bar{\eta}'_1 = \bar{L}'/\beta$ where $\bar{L}'$ is the length they measure.

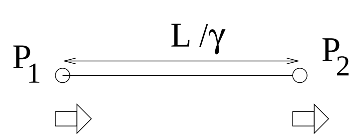

FIG. 4: O measures a Lorenz contraction of the distance between P_1 and P_2.

In C's frame, which is at rest with respect to the ring, the length along the axis is simply L. C sees the rocket pass with speed β and determines that it reached point P_2 a time L/β after it coincided with point P_1. We can equate spacetime intervals to derive the Lorenz contrac-

tion:

$$\begin{aligned} -(\bar{\eta}_2' - \bar{\eta}_1')^2 + (\bar{x}_2' - \bar{x}_1')^2 &= -(\bar{\eta}_2 - \bar{\eta}_1) + (\bar{x}_2 - \bar{x}_1)^2 \\ -\frac{\bar{L}'^2}{\beta^2} &= -\frac{L^2}{\beta^2} + L^2 \end{aligned} \quad (3)$$

which yields $\bar{L}' = L/\gamma$. Observer O believes the ring is contracted relative to C's measure. However, the emission of last scattered light from the surface of last scattering constitutes a slightly different measurement. In that case, O sees light emitted from point P_2 first and only after the ring has continued to move for a time $|\Delta\eta'| = \gamma\beta L$ (as derived in eqn. (1)) does P_1 emit last scattered light. Consequently the point P_1 has moved an additional distance $\gamma\beta^2 L$ on top of the intrinsic separation of L/γ giving a net separation of

$$\gamma\beta^2 L + L/\gamma = \gamma L \quad (4)$$

and confirming the first derivation. O perceives the ring to be wider along the direction of motion than it is perpendicular to the direction of motion.

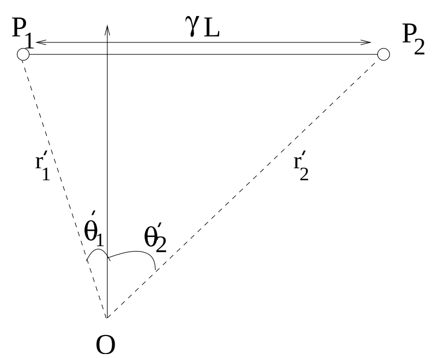

FIG. 5: The points P_1 and P_2 subtend angles θ_1' and θ_2' respectively.

To determine the angles subtended by the points P_1 and P_2 as measured by O (figure 5), consider the two events, emission of light from P_2 and the receipt of that light. In the C frame, light is emitted at η_2 and received at η_0 so that $\eta_0 - \eta_2 = r$ where r is the distance the light has traveled. These two events are separated by $x_0 - x_2 = -r\sin\theta = -L/2$, where θ is the angular radius of the circle in the stationary frame of C.

From the Lorenz transformation, the separation of the two events $\Delta\eta' = \eta_0' - \eta_2'$ in time and $\Delta x_2' = x_0' - x_2'$ in space are given by

$$\begin{aligned} \Delta\eta_2' &= \gamma(1 + \beta\sin\theta)r = r_2' \\ \Delta x_2' &= -\gamma(\sin\theta + \beta)r = -r_2'\sin\theta_2' \end{aligned} \quad (5)$$

where r_2' is the distance light travels in the O frame and θ_2' is the angle subtended by point P_2, as measured from the vertical.

Similarly for point P_1 it follows that

$$\begin{aligned} \Delta\eta_1' &= \gamma(1 - \beta\sin\theta)r = r_1' \\ \Delta x_1' &= \gamma(\sin\theta - \beta)r = r_1'\sin\theta_1' \end{aligned} \quad (6)$$

where θ_1' is the magnitude of the angle P_1 subtends from the verticle as drawn in figure 5. Notice that $\Delta\eta_1' - \Delta\eta_2' = \eta_2' - \eta_1' = -\gamma\beta L$ and $\Delta x_1' - \Delta x_2' = x_2' - x_1' = \gamma L$ confirming the results of eqn. (1).

It follows from eqn. (5) that

$$\sin\theta_2' = \frac{(\sin\theta + \beta)}{(1 + \beta\sin\theta)} \quad . \quad (7)$$

If we assume that $\theta_2' = \theta + \delta_2$ where δ_2 is small, then the angle can be approximated by $\sin\theta_2' \simeq \sin\theta + \delta_2\cos\theta$. Eqn. (7) then yields

$$\delta_2 \simeq \frac{\beta\cos\theta}{(1 + \beta\sin\theta)} \quad . \quad (8)$$

The angle subtended by P_2 is larger than in the comoving frame by a piece proportional to β. Similarly, the angle subtended by P_1 is smaller by $\theta_1' = \theta - \delta_1$,

$$\delta_1 \simeq \frac{\beta\cos\theta}{(1 - \beta\sin\theta)} \quad . \quad (9)$$

The center of the circle is displaced by $\delta \sim \beta$.

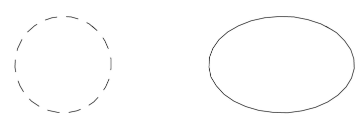

FIG. 6: Observer C measures the dotted-line circle centered on $x = 0$ while observer O measures the solid-line ellipse displaced by $\gamma\beta r$. The circle is displaced as well as distored in shape. In the figure r is taken to be $2L$ for illustration and $\beta = 3/4$.

In C's frame, the circles of figure 2 lie in the $x - z$ plane and are parameterized by the 4-vector $(\eta_* - \eta_0, x - x_0, y - y_0, z - z_0) = (-r, L\cos\phi/2, 0, L\sin\phi/2)$. In O's frame, the shape of the intersection of the copies of the surface of last scattering is parameterized by

$$\begin{aligned} x' &= \gamma\left(\frac{L\cos\phi}{2}\right) + \gamma\beta r \\ z' &= \frac{L}{2}\sin\phi \end{aligned} \quad (10)$$

and is a displaced ellipse as shown in figure 6. Notice that the distortion to the shape is proportional to γ and so is second order in β while the displacement is first order in β and is therefore the bigger effect. For circles that are not perpendicular to the direction of motion the shape will be slightly different from a perfect ellipse.

In the boosted frame, the center of one oval is offset by δ from the vertical and its pair is not 180^o further on at an angle of $180^o + \delta$ but instead is centered at $180^o - \delta$. The center of the pair is off by 2δ as illustrated in figure 7.

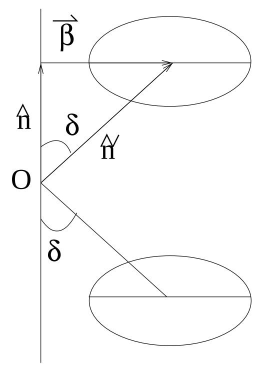

FIG. 7: In figure 2 observer C sees one circle in front of him and one behind him. Unlike C, the Lorenz boosted observer O does not see one cirlce in front of him and one behind him.He sees the center of both ovals displaced by $\delta \sim \beta$ as drawn. Notice the oval pairs are opposite each other and face on but they are not on either side of the observer O.If he were to find an oval in direction δ and were to look for its pair in the direction $180^o + \delta$, he would miss the pair by an angle $\sim 2\delta \sim 2\beta$ (for small δ and β).

The above calculation applies to a specific geometry. More generally, given any point on the surface of last scattering $(\eta_* - \eta_o, r\hat{n}) = r(-1, \hat{n})$, the direction to that point is Lorenz transformed to

$$\hat{n}' = \frac{\hat{n} + \left[(\gamma - 1)\hat{\beta} \cdot \hat{n} + \gamma\beta\right]\hat{\beta}}{\gamma(1 + \vec{\beta} \cdot \hat{n})} \quad . \tag{11}$$

(The data could be corrected by applying the inverse of this transformation to the map, that is, if we were in uniform motion.) In particular, if $\hat{n}$ is the direction to the center of a circle, then to lowest order in β for the case of $\hat{n} \cdot \vec{\beta} = 0$, the center wil be displaced to

$$\hat{n}' = \hat{n} + \vec{\beta} \tag{12}$$

If we were to look for an antipodal pair in the opposite direction, $-\hat{n}$, the pair would also be displaced to $\hat{n}'_{pair} \simeq -\hat{n} + \vec{\beta} = -\hat{n}' + 2\vec{\beta}$ which is not 180^o oposite as illustrated in figure 7.

The effect is largest when $\hat{n} \cdot \hat{\beta} = 0$ as in figure 2 and the effect vanishes when $\hat{n} \cdot \hat{\beta} = 1$. For $\beta \sim 1.23 \times 10^{-3}$, the maximum angular difference in the location of a pair from where one expects is

$$2\beta \sim 0.14^o \quad . \tag{13}$$

The deviation is just below WMAP's angular resolution which at best is $< 0.25^o$ in the 90 GHz channel and at worst is $\sim 0.93^o$ in the 22 GHz channel. The stretching and displacement of circles-in-the-sky would also be relevant for the future *Planck Surveyor* which aims for an angular resolution of around several arc-minutes. Still, if the data is smoothed before scanned, the aberration may be below the resolution of circle searches. It will be something to bear in mind for future analysis.

Distortions due to the motion of our planet and galaxy could effect any statistical, pattern-based search, not just circles-in-the-sky. There are other modifications to consider as well, such as the complication that the earth is not in uniform motion and the extension to a curved space. This calculation is intended to draw out the general issue.

An intriguing possibility that is highlighted here and in earlier articles on special relativistic effects in a finite universe [10, 11], involves compactifying space*time* and not just space. The universe is a (3+1)-dimensional spacetime and in principle we should consider the geometry of this four-dimensional manifold. Of course, we have little idea how to go about this as much due to difficulties of interpretation as anything else. It is a reminder that whatever underlying principle determines the creation of the universe and its topology will shape our future ideas on the nature of space and time.

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